

Systèmes Hybrides

Formation sur les Systèmes à Événements Discrets (SED)

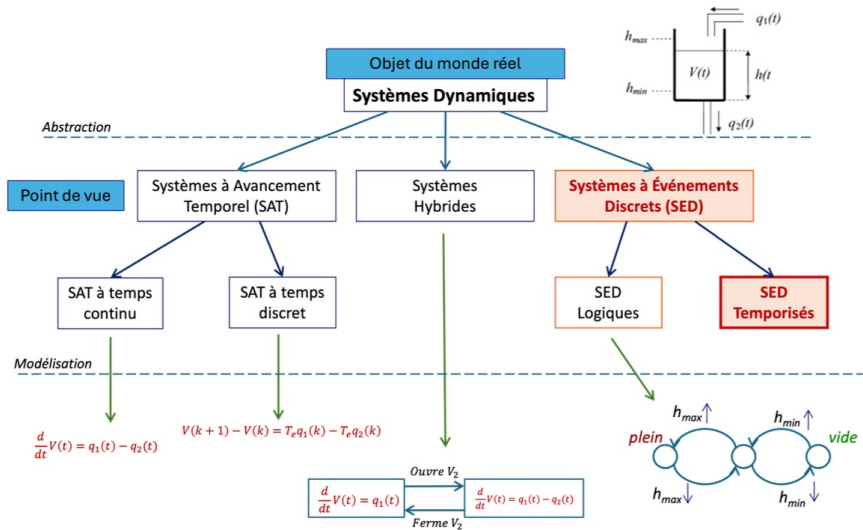
2^e édition
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Nantes



Société d'Automatique,
de Génie Industriel & de Productique



Introduction



- 1 Introduction : Examples of Hybrid Systems
- 2 Modeling of Hybrid Systems
- 3 Analysis of Switched Systems
- 4 Simulation of Switched Systems
- 5 Control of Switched Systems
- 6 Conclusions

1. Examples of Hybrid Systems

- Automatic Gearbox (vehicle)
- Bipedal Robots
- Electronic Switching Circuit
- Nuclear Plant
- etc.

Hybrid Systems =

Continuous State + Switching

⇒ **Switched Systems** is a particular class of **Hybrid Systems**

Example 1 : Automatic Gearbox (Vehicle)

- θ = position
- $\omega = \dot{\theta}$ = velocity



Automatic
Transmission

VS



Manual
Transmission

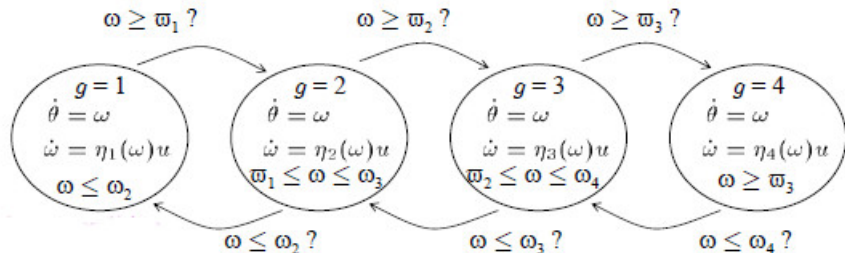


Figure – Automatic Gearbox

Example 2 : Bipedal Robots

- Walking Robot

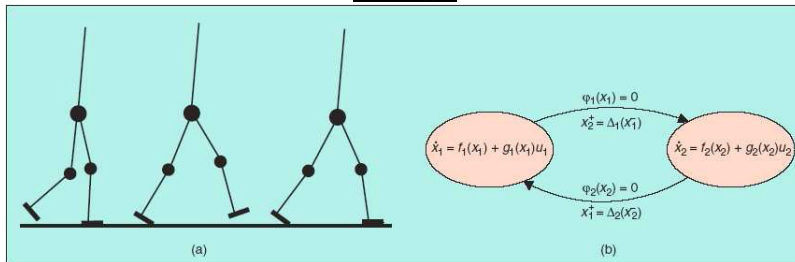


Figure – Bipedal Robot

Example 3 : Electronic Switching Circuit

- $\dot{x} = A_i x(t) + B_i u$
where $A_i \in \{A_1, A_2, A_3, A_4\}$
- $x = [x_1, x_2, x_3] = [V_{C1}, V_{C2}, \text{current}]$
- $u = [u_1, u_2, u_3] \in \{0, 1\}^3$

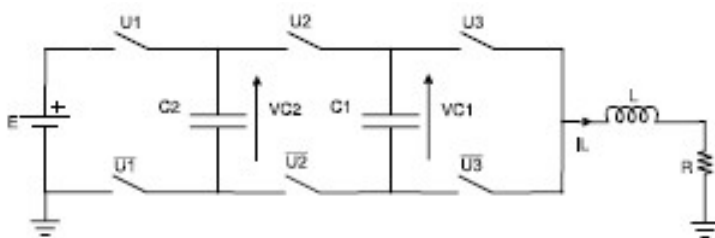


Figure – Flying Capacitor (Riedinger et al., Automatica 2010)

Example 4 : Nuclear Plant (1)

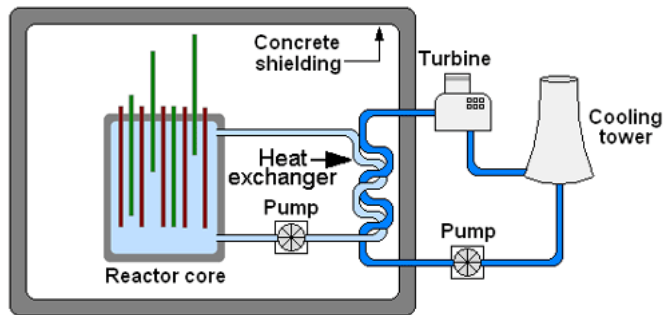


Figure – Nuclear Plant

Example 4 : Nuclear Plant (2)

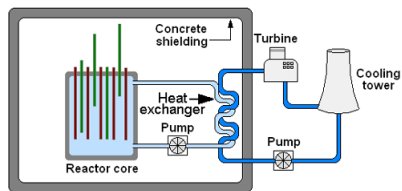


Figure – Nuclear Plant

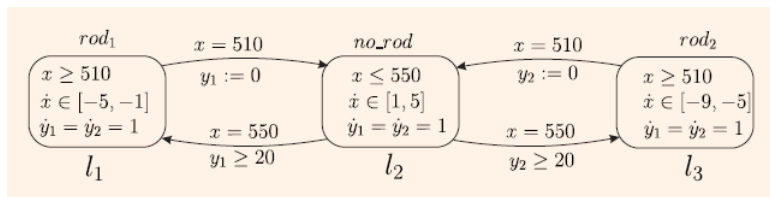


Figure – Hybrid Behavior

Hybrid Systems : Examples

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- Bipedal Robots
- Electronic Switching Circuit
- Nuclear Plant
- etc.

Hybrid Systems =

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2. Modeling of Hybrid Systems

- From Computer Science :
 - Hybrid Automata
 - Hybrid Petri Nets
 - Statecharts

- From Control :
 - Differential Equations
 - State Equations with index
 - Set of Indexed Vector Fields

Question :

Hybrid Systems : Computer Science or Control ?

Computer Science vs. Control [Krogh 1995]

- **Computer Science** :
 - **Specification** : Define syntax and semantics as a program
 - **Verification** : "program = specification" ?
(e.g. reachability analysis; model checking)

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- **Analysis** : of stability
- **Control Design** : Synthesize a controller so that
"system + controller = specification"

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Hybrid Systems

Switched **Linear** Systems (Electronic Switching Circuit)

- $\dot{x} = A_i x(t) + B_i u$
where $A_i \in \{A_1, A_2, A_3, A_4\}$
- $x = [x_1, x_2, x_3] = [V_{C1}, V_{C2}, \text{current}]$
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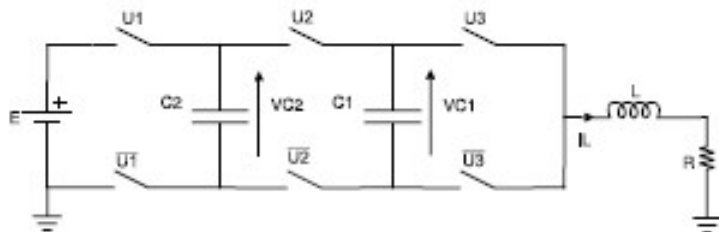


Figure – Flying Capacitor (Riedinger et al., Automatica 2010)

Switched **Non-Linear** Systems.

$$\dot{x}(t) = f_{\sigma(t)}(x, u, t)$$

where :

- f_i are vector fields
- x is the state
- u is the exogenous input
- t is the time
- $\sigma(t) : [0, t_f] \rightarrow \mathcal{Q} \in \{0, 1, 2, \dots, q\}$ is a *piecewise constant* function of time = the **switching signal**

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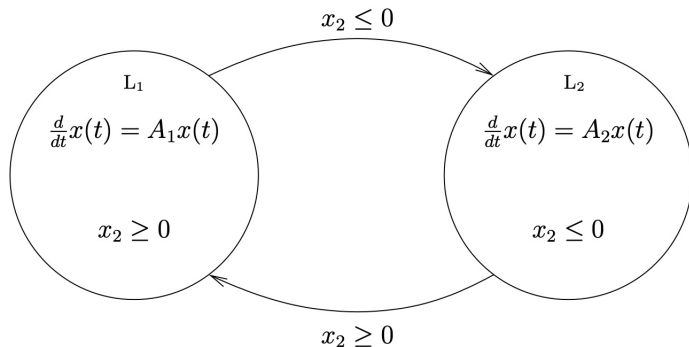
cf. [Liberzon 2003]

Two basic problems :

- Stability of switched systems under **arbitrary switchings**
- Stability of switched systems under **constrained switchings**

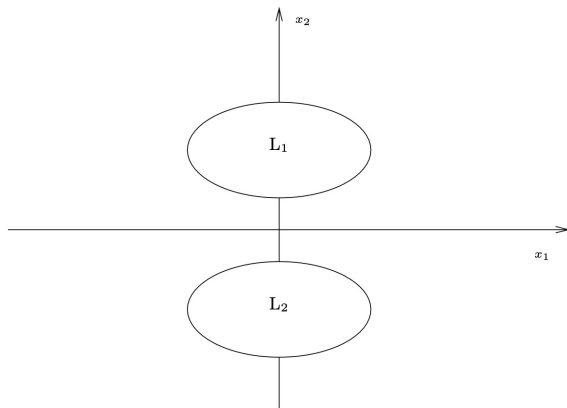
Stability of Switched Systems

Switching between $A_1 = \begin{bmatrix} -0.1 & 2 \\ -5 & -0.1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} -0.1 & 5 \\ -1 & -0.1 \end{bmatrix}$



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cf. [Lin and Antsaklis, TAC'08]

- Stability under arbitrary switchings

- - Common quadratic Lyapunov functions
- - Special cases
 - state matrices that are pairwise commutative
 - state matrices that are symmetric
 - switched *normal* systems $A_q A_q^T = A_q^T A_q$
- - Necessary and Sufficient condition [Lin and Antsaklis, CDC'04]
- - Converse Lyapunov Theorem [Dayawansa and Martin, TAC'99]

- Stability under **constrained switchings**
 - Time-domain : dwell-time, average dwell-time, etc. [Hespanha and Morse, CDC'99], [Hespanha, TAC'04]
 - State space : abstractions from partitions of the state space
 - Multiple Lyapunov functions (MLF) [Peleties, deCarlo ACC'91]

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- From Computer Science :
 - Hybrid Automata \Rightarrow **HyTech** [T. Henzinger]
 - Hybrid Petri Nets \Rightarrow many Petri Nets Simulation tools.
 - Statecharts \Rightarrow **Matlab/StateFlow** [Mathworks]

- From Control :
 - Differential Equations \Rightarrow Modelica (Object Oriented)
 - State Equations with index \Rightarrow **Matlab/Simulink** [Mathworks]
 - Set of Indexed Vector Fields

Question : What is the best Simulator for Hybrid Systems ?

Exercise N.1 (1/3)

Consider a switched system whose behavior is defined by the following PWA (piecewise affine system) model :

$$\dot{x} = \begin{cases} A_1 x & \text{if } x_1 x_2 \geq 0 \\ A_2 x & \text{if } x_1 x_2 < 0 \end{cases}$$

where $x = [x_1 x_2]^T$, $A_1 = \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix}$

Exercise N.1 (2/3)

Switching between $A_1 = \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix}$

1. Prove that each sub-system A_1 and A_2 is **stable**, respectively.
2. **Simulate** with MATLAB the output ($y(t) = x(t)$) and the phase portrait of **sub-system** A_1
3. **Simulate** with MATLAB the output ($y(t) = x(t)$) and the phase portrait of **sub-system** A_2
4. **Simulate** with MATLAB the output ($y(t) = x(t)$) and the phase portrait of the **switched system**. (you have to chose 2 different initial conditions. One initial condition for which A_1 is active first, and another initial condition for which A_2 is active first.)

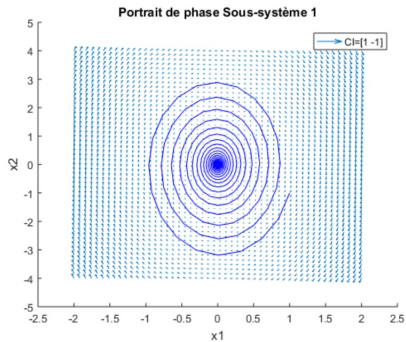
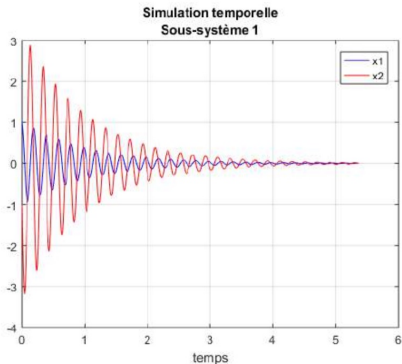
Exercise N.1 (3/3)

Switching between $A_1 = \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix}$

5. **Prove** that the function $V(x) = x_1^2 + x_2^2$ is a **common Lyapunov function** for sub-system A_1 and sub-system A_2 .
 \Rightarrow (**Hint** : Prove that a) $V(0)=0$, b) $V(x) > 0$ and c) $\frac{\partial V}{\partial x} \dot{x} < 0$).
6. Conclusions ?

Simulation of Switched Systems : with MATLAB

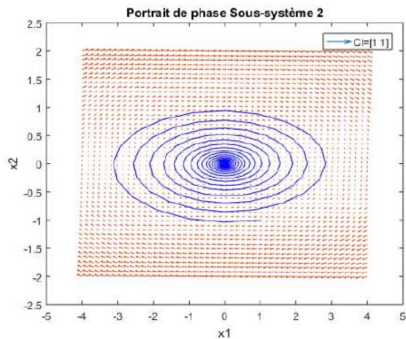
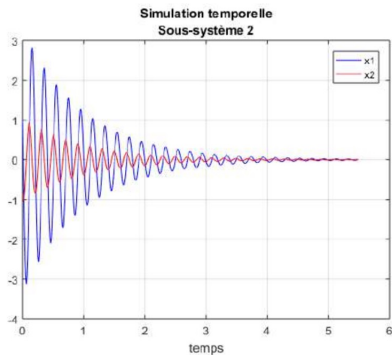
Solution (Exercise N.1) :
Sub-system A_1 alone



Simulation of Switched Systems : with MATLAB

Solution (Exercise N.1) :

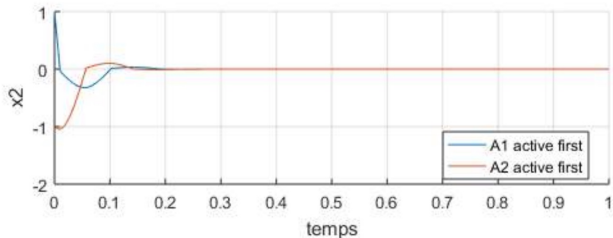
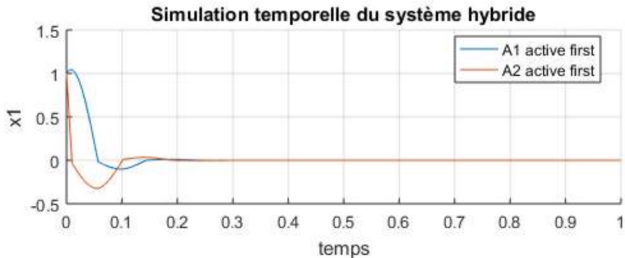
Sub-system A_2 alone



Simulation of Switched Systems : with MATLAB

Solution (Exercise N.1) :

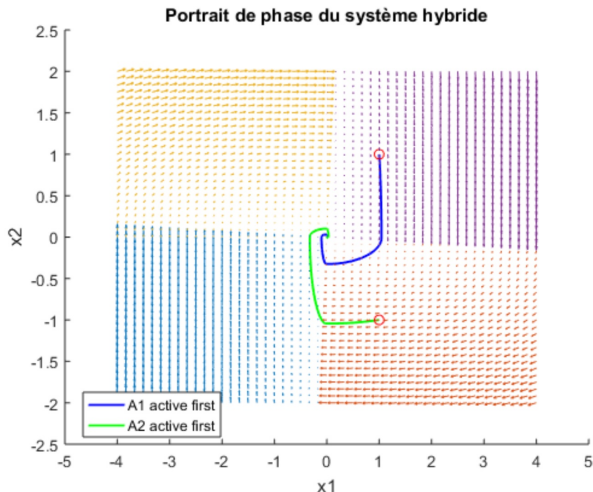
Switch(A_1, A_2)



Simulation of Switched Systems : with MATLAB

Solution (Exercise N.1) :

Switch(A_1, A_2)



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Control of Switched Systems : LQ Design (with MATLAB)

Exercise N.2 (1/2)

Given the (controlled) switched system :

$$dx(t)/dt = A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)}, \quad x(0) = x_0$$

$$\text{with } A_1 = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The problem is to **minimize** the following cost function :

$$\min_{\sigma} \frac{1}{2} \int_0^{\infty} x^T(t) \tilde{Q}_{\sigma(t)} x(t) dt$$

subject to $\dot{x}(t) = A_{\sigma(t)}x(t) - B_{\sigma(t)}K_{\sigma(t)}x(t)$ where $\tilde{Q}_i = Q + K_i^T R_i K_i$ and where matrices (R_i, Q_i) and K_i correspond to an LQ design for the subsystem (A_i, B_i) , $i = 1, 2$.

Control of Switched Systems : LQ Design (with MATLAB)

Exercise N.2 (2/2)

1. Calculate the static gains K_1 and K_2 . Prove that $K_1 = K_2 = 0.2361$.

Remarks

- The pairs (A_1, B_1) and (A_2, B_2) are not stabilizable, respectively, because $ctrb(A_1, B_1)$ is not of full rank (rank=1 instead of 2). The same for $ctrb(A_2, B_2)$.
 - Solution : Assume $sys1 = ss(A_1, B_1, C_1, D_1)$ with $C_1 = eye(2)$ and $D_1 = 0$. Then calculate $msys1 = minreal(sys1)$ which is the controllable part of the first system. Then K_1 is obtained using the LQR command applied to $msys1$. Do the same method for the second system to calculate K_2 .
2. Then simulate the switched system defined by $\dot{x} = (A_i - B_i K_i)x, \quad i = 1, 2.$
 3. Conclusions

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Conclusions (1/2)

Summary

- It was just an Introduction to Hybrid Systems
- **Computer Science** vs. **Control** [Krogh 1995]
- **Control** :
 - **Analysis** : of stability
 - **Control Design** : Synthesize a controller so that "system + controller = specification"
- Exercises (**Simulation, Control Design**) with MATLAB

Some Promising Approaches (from the DES point of view) :

- **Switching Max-Plus Systems**
 - [van den Boom, de Schutter 2006, 2008]
 - [Zorzenon, Komenda, Raisch 2022, 2023]
 - [Animobono, Scaradozzi, Zattoni, Perdon, Conte 2022]
 - etc.

Références bibliographiques

Main References



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Birkhauser, Boston, 2003.



H. Lin and P.J. Antsaklis,
"Stability and Stabilizability of Switched Linear Systems : A Survey of
Recent Results,"
IEEE Trans. on Automatic Control, vol. 54 (2), Feb 2009.

(and the references therein)

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Intervenant.e.s : Naly Rakoto

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