Réseaux de Petri temporels - Espace d'états

Formation Systèmes à Evénements Discrets

1ère édition
Janvier 2024

Société d'Automatique,
de Génie Industriel & de Productique

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Plan

1. Abstraire l'espace d'états
2. Graphe des Zones
   - Présentation de l'algorithme
   - Application sur la séquence : $\mathcal{Z}_1 \xrightarrow{t_2} \mathcal{Z}_2 \xrightarrow{t_3} \mathcal{Z}_3 \xrightarrow{t_2} \mathcal{Z}_4 \xrightarrow{t_3} \mathcal{Z}_5$
   - Terminaison
3. Outils

Abstraire l'espace d'états

Problème de la vérification de modèles

$\Rightarrow$ Explorer l'espace d'états

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Problème

L'espace d'états d'un TPN est infini (en général)

Problème de la vérification de modèles

$\Rightarrow$ Explorer l'espace d'états

Problème

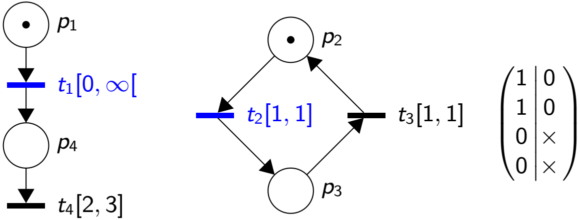
L'espace d'états d'un TPN est infini (en général)

$\Rightarrow$ Regrouper les états en classes d'équivalence (abstraction)

Exploration de l'espace d'état

- par simulation

- par simulation



Exploration de l'espace d'état

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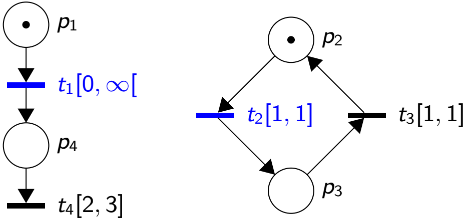
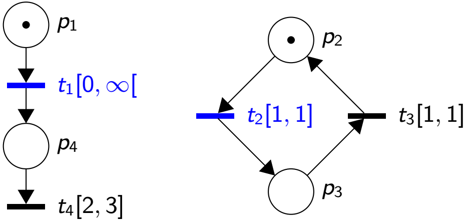


$$\left(\begin{array}{c|c} 1 & 0 \\ 1 & 0 \\ 0 & \times \\ 0 & \times \end{array}\right) \xrightarrow{0} \left(\begin{array}{c|c} 1 & \delta \\ 1 & \delta \\ 0 & \times \\ 0 & \times \end{array}\right)$$

- par simulation



$$\begin{pmatrix} 1 & \vert & 0 \\ 1 & \vert & 0 \\ 0 & \vert & \times \\ 0 & \vert & \times \end{pmatrix} \begin{matrix} \xrightarrow{0} \\ \xrightarrow{0.7} \end{matrix} \begin{pmatrix} 1 & \vert & \delta \\ 1 & \vert & \delta \\ 0 & \vert & \times \\ 0 & \vert & \times \end{pmatrix}$$

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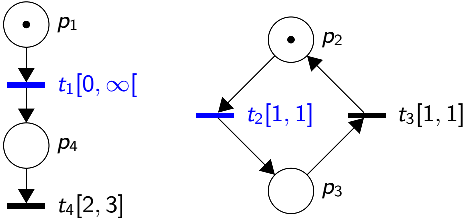

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \times \\ 0 & \times \end{pmatrix} \begin{array}{c} \xrightarrow{0} \\ \xrightarrow{0.7} \\ \dots \\ \xrightarrow{1} \end{array} \begin{pmatrix} 1 & \delta \\ 1 & \delta \\ 0 & \times \\ 0 & \times \end{pmatrix}$$

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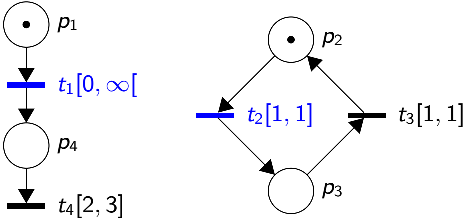
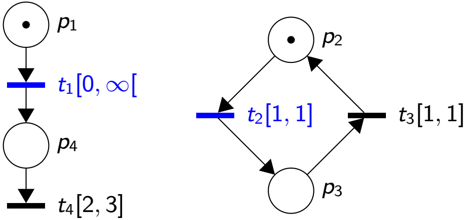


$$\left(\begin{array}{c|c} 1 & 0 \\ 1 & 0 \\ 0 & \times \\ 0 & \times \end{array}\right) \quad \begin{array}{c} \xrightarrow{0} \\ \xrightarrow{0.7} \\ \dots \\ \xrightarrow{1} \end{array} \quad \left(\begin{array}{c|c} 1 & \delta \\ 1 & \delta \\ 0 & \times \\ 0 & \times \end{array}\right)$$

$\rightarrow$ Infinité de branchements, d’états

- par simulation



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$\rightarrow$ Infinité de branchements, d'états

- symbolique

Exploration de l'espace d'état

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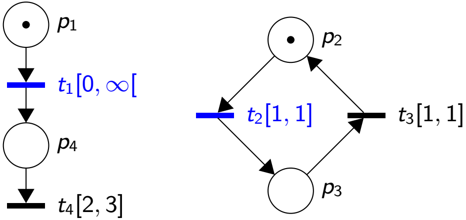

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \times \\ 0 & \times \end{pmatrix} \begin{array}{c} \xrightarrow{0} \\ \xrightarrow{0.7} \\ \dots \\ \xrightarrow{1} \end{array} \begin{pmatrix} 1 & \delta \\ 1 & \delta \\ 0 & \times \\ 0 & \times \end{pmatrix}$$

$\rightarrow$ Infinité de branchements, d'états

- symbolique
  - regroupement d'états :

$$\begin{pmatrix} 1 \\ 1, x_1 = x_2 = 0 \\ 0 \end{pmatrix}$$

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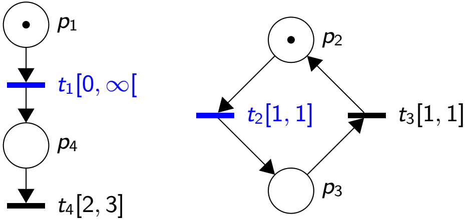

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$$\begin{pmatrix}1\\1, x_1 = x_2 = 0\\0\end{pmatrix} \xrightarrow{\delta}$$

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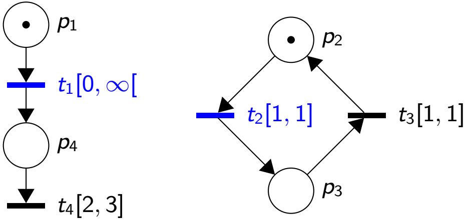

$$\begin{pmatrix} 1 & \vline & 0 \\ 1 & \vline & 0 \\ 0 & \vline & \times \\ 0 & \vline & \times \end{pmatrix} \quad \begin{matrix} \xrightarrow{0} \\ \xrightarrow{0.7} \\ \dots \\ \xrightarrow{1} \end{matrix} \quad \begin{pmatrix} 1 & \vline & \delta \\ 1 & \vline & \delta \\ 0 & \vline & \times \\ 0 & \vline & \times \end{pmatrix}$$

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$$\begin{pmatrix} 1 \\ 1, x_1 = x_2 = 0 \\ 0 \end{pmatrix} \xrightarrow{\delta} \begin{pmatrix} 1 \\ 1, x_1 = x_2 \in [0, 1] \\ 0 \end{pmatrix}$$

Exploration de l'espace d'état

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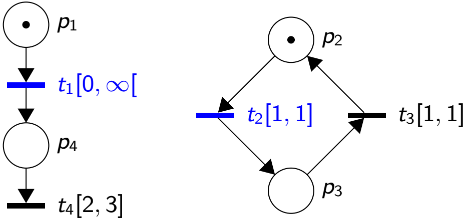

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$\rightarrow$ Graphe des classes [Berthomieu and Menasche, 1983, Berthomieu and Diaz, 1991]

$\rightarrow$ Graphe des regions [Dill, 1989, Alur et al., 1990]

$\rightarrow$ Graphe des zones [Dill, 1989, Gardey et al., 2003]

Calcul de l'espace d'états

État symbolique

Definition (État symbolique)

$\mathcal{Q} = (M, \mathcal{V})$

- M un marquage,
- $\mathcal{V}$ ensemble de valuations pour lequel M existe.

Plan

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- Terminaison

3 Outils

Présentation de l'algorithme

Computation of the state space. Initial zone

Step 1

$\mathcal{Z}_0 = (M_0, Z_0)$
Computation of the states reachable by time elapsing (futur)

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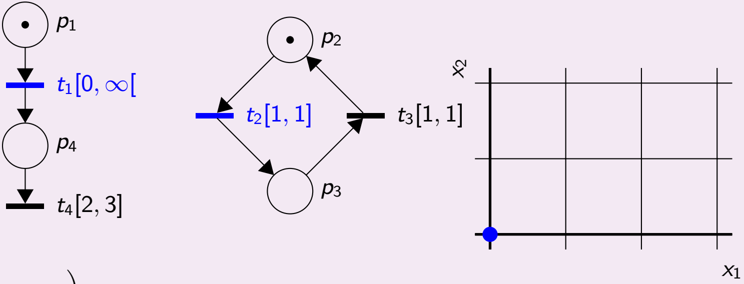
Computation of the states reachable by time elapsing (futur)

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, x_1 = x_2 = 0 \right)$$

Computation of the state space. Initial zone
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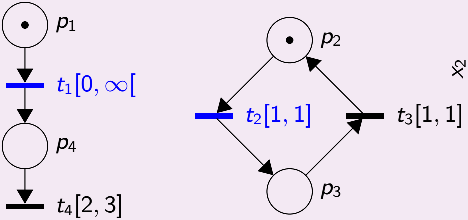

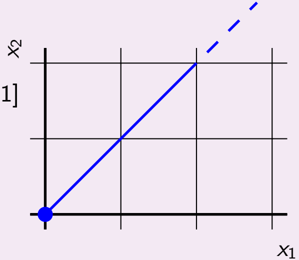

$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, x_1 = x_2 = 0 \right) \rightarrow \left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} x_1 \geq 0 \\ x_2 \geq 0 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right)$$

Futur

Computation of the state space. Initial zone

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Computation of the states reachable by time elapsing (futur)

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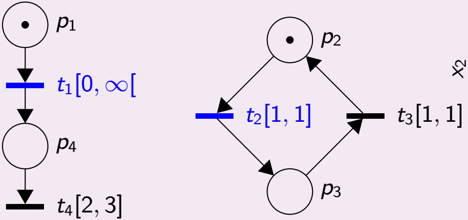
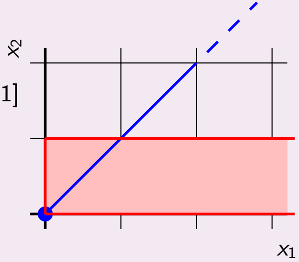




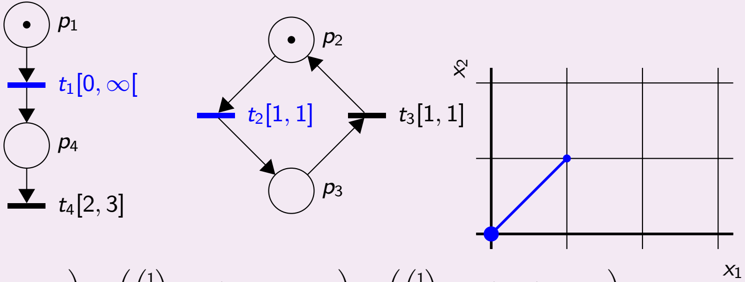
$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, x_1 = x_2 = 0 \right) \rightarrow \left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} x_1 \geq 0 \\ x_2 \geq 0 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right)$$

Futur $\cap (x_1 \leq \infty) \cap (x_2 \leq 1)$

$\mathcal{Z}_0 = (M_0, Z_0)$

Computation of the states reachable by time elapsing (futur)

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, x_1 = x_2 = 0 \right) \rightarrow \left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} x_1 \geq 0 \\ x_2 \geq 0 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right) \rightarrow \left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right) = Z_0$$

Futur $\cap (x_1 \leq \infty) \cap (x_2 \leq 1) \rightarrow Z_0$

Fireability
Step 2

Checking t_1 is fireable from $\mathcal{Z}_0 = (M_0, Z_0)$.

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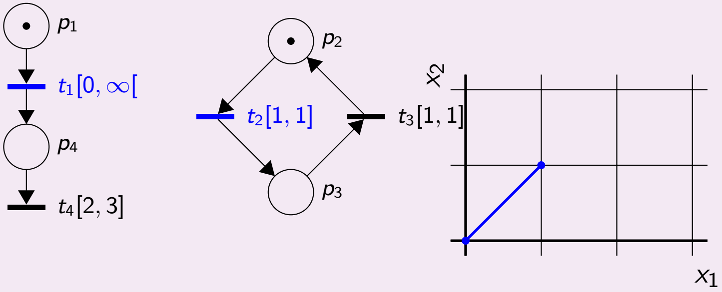
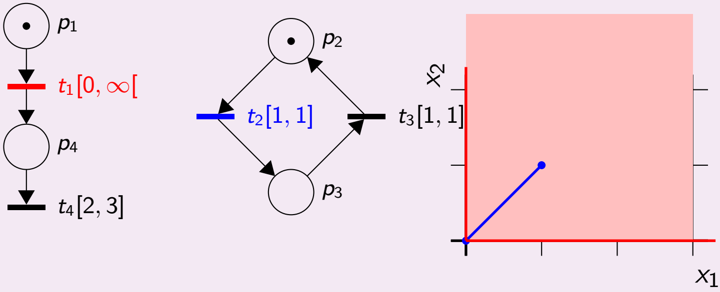


Fireability
Step 2

Checking t_1 is fireable from $\mathcal{Z}_0 = (M_0, Z_0)$.

Example



$$(Z_0 \cap x_1 \geq \alpha_1) \neq \emptyset$$

True : t_1 is fireable from $\mathcal{Z}_0$

Fireability

Step 2

Checking t_2 is fireable from $\mathcal{Z}_0 = (M_0, Z_0)$.

Fireability

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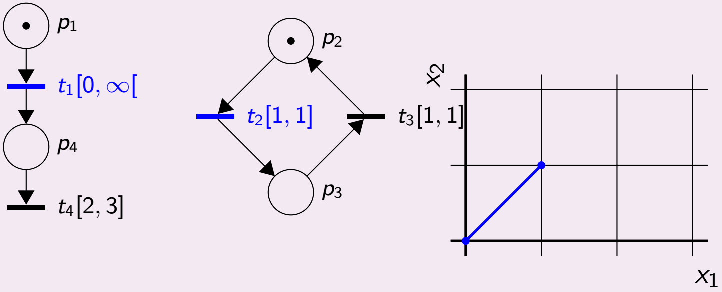
Checking t_2 is fireable from $\mathcal{Z}_0 = (M_0, Z_0)$.

Example



Fireability
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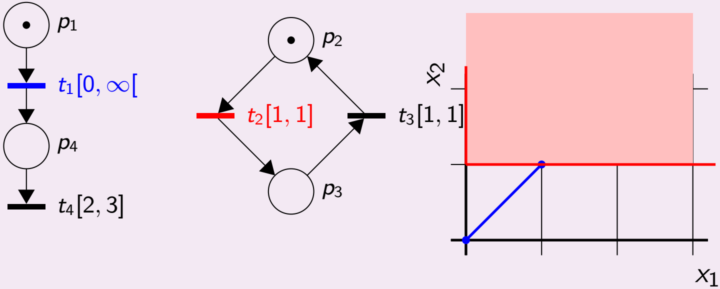
Checking t_2 is fireable from $\mathcal{Z}_0 = (M_0, Z_0)$.

Example



$$(Z_0 \cap x_2 \geq \alpha_2) \neq \emptyset$$

True : t_2 is fireable from $\mathcal{Z}_0$

Computation of next

Step 3

Firing of $t_1 \rightarrow next\,((M_0, Z_0), t_1) = (M_1, Z_1)$.

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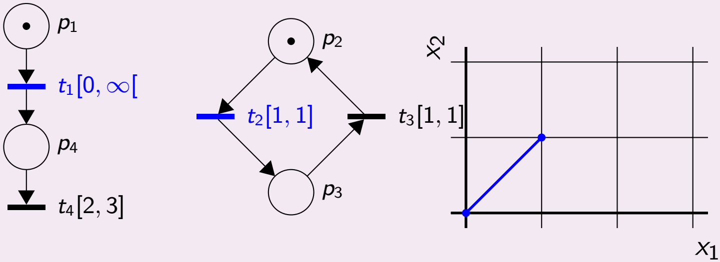
Firing of $t_1 \rightarrow next\left((M_0, Z_0), t_1\right) = (M_1, Z_1)$.

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right)$$

Computation of next

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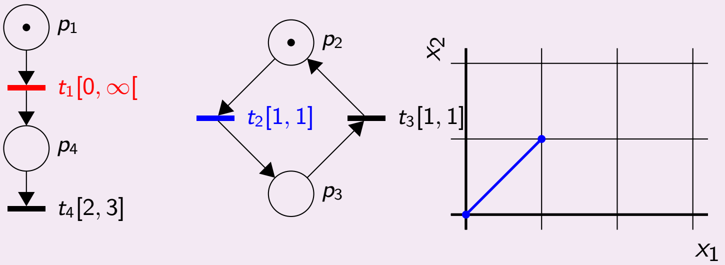
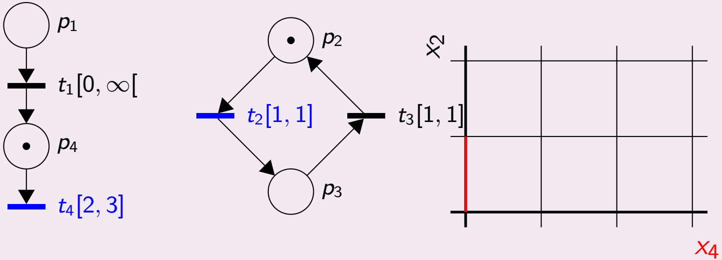


$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right) \xrightarrow{t_1}$$

$M_1 = M_0 - {}^\bullet t_1 + t_1^\bullet$, Firing from $Z_0 \cap (x_1 \geq \alpha_1)$

Firing of $t_1 \rightarrow next\,((M_0, Z_0), t_1) = (M_1, Z_1)$.

Example

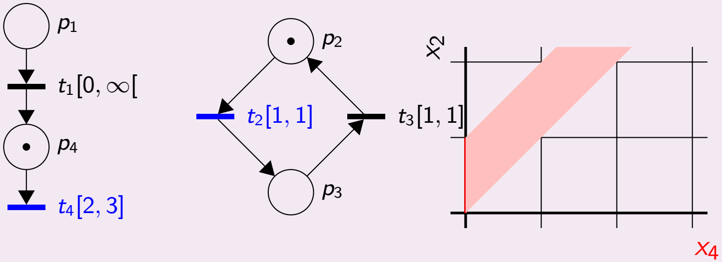


$$\left(\begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{array}{l} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{array}\right) \xrightarrow{t_1} \left(\begin{pmatrix}0\\1\\0\\1\end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 0 \\ 0 \leq x_2 - x_4 \leq 1 \end{array}\right)$$

Eliminate x_1 (Fourier-Motzkin method) and add $x_4 = 0$

Firing of $t_1 \rightarrow next\,((M_0, Z_0), t_1) = (M_1, Z_1)$.

Example



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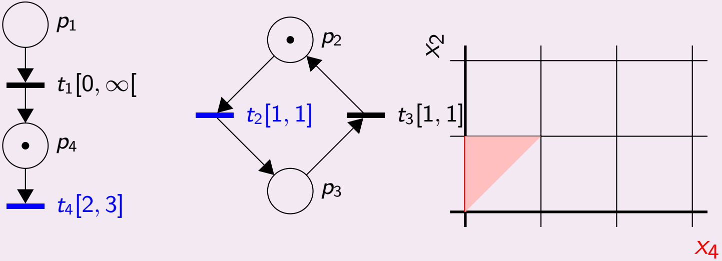
Compute the futur

Computation of next

Step 3

Firing of $t_1 \rightarrow next\,((M_0, Z_0), t_1) = (M_1, Z_1)$.

Example



$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{matrix} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{matrix} \right) \xrightarrow{t_1} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 3 \\ 0 \leq x_2 - x_4 \leq 1 \end{matrix} \right)$$

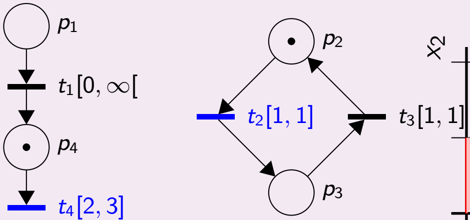
Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$

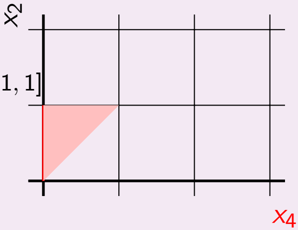
Computation of next

Step 3

Firing of $t_1 \rightarrow next\,((M_0, Z_0), t_1) = (M_1, Z_1)$.

Example





$$\left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{array}{l} 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1 \\ 0 \leq x_1 - x_2 \leq 0 \end{array} \right) \xrightarrow{t_1} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 3 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) = \mathcal{Z}_1$$

Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$ in canonical form

The algorithm of next

Next(Z,t)

Let t a transition with the firing interval $[\alpha, \beta]$ fireable from (M_i, Z_i). The computation of the successor of (M_i, Z_i) by the firing of t is $next((M_i, Z_i), t) = (M_j, Z_j)$ where Z_j is computed as follows:

- compute the firing space of the transition t : $Z_i \cap (x_t \geq \alpha)$ where x_t is the clock associated with t
- eliminate x_t (for example by using Fourier-Motzkin method)
- add (or reset) the clocks of the newly enabled transitions: x_{new} and for all other clocks x_{old} (with $min \leq x_{old} \leq max$), add the new diagonal constraints $min \leq x_{old} - x_{new} \leq max$)
- compute the futur (time elapsing)
- add the constraints $x \leq \beta$
- compute the canonical form

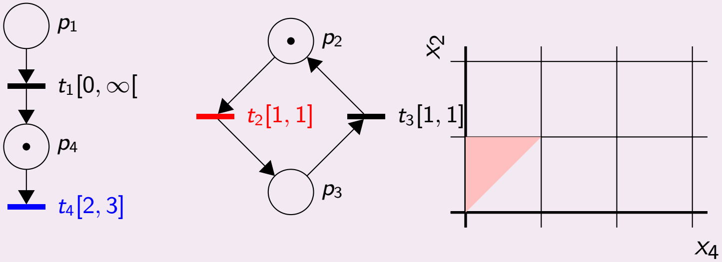
Application sur la séquence :

$$\mathcal{Z}_1 \xrightarrow{t_2} \mathcal{Z}_2 \xrightarrow{t_3} \mathcal{Z}_3 \xrightarrow{t_2} \mathcal{Z}_4 \xrightarrow{t_3} \mathcal{Z}_5$$

Firing of $t_2 \rightarrow \mathit{next}((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Firing of $t_2 \to next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

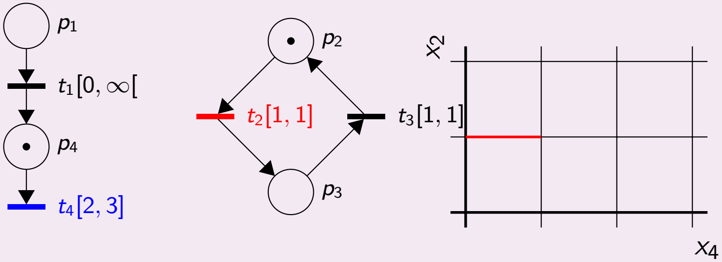
Example



$$\mathcal{Z}_1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right)$$

Firing of $t_2 \to next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Example

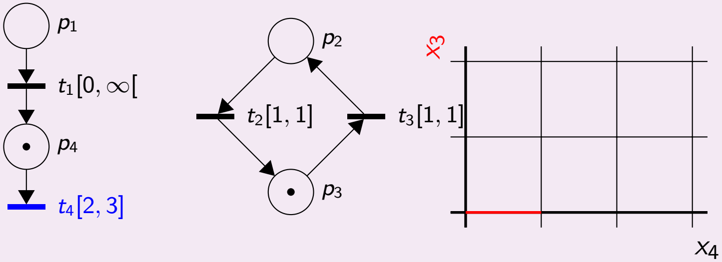


$$\mathcal{Z}_1 \cap x_2 \geq 1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 1 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) \xrightarrow{t_2}$$

$$(M_1 - {}^\bullet t_2 + t_2^\bullet,\ Z_1 \cap x_2 \geq \alpha_2)$$

Firing of $t_2 \to next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Example

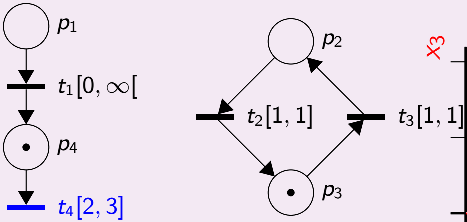


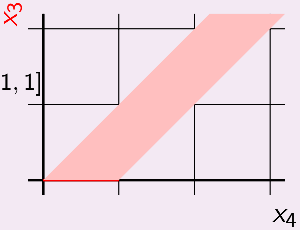
$$\mathcal{Z}_1 \cap x_2 \geq 1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 1 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 0 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right)$$

Eliminate x_2 (Fourier-Motzkin method) and add $x_3 = 0$

Firing of $t_2 \rightarrow next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Example



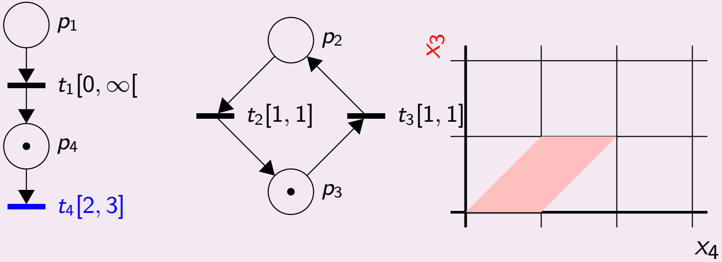


$$\mathcal{Z}_1 \cap x_2 \geq 1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 1 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \\ 0 \leq x_4 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right)$$

Compute the futur

Firing of $t_2 \rightarrow next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Example

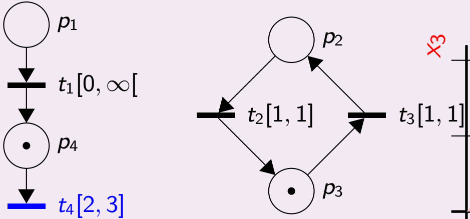


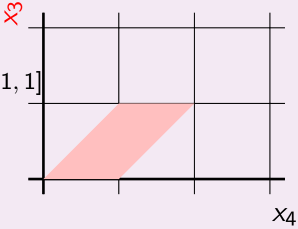
$$\mathcal{Z}_1 \cap x_2 \geq 1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 1 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 3 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right)$$

Compute the futur $\cap\, x_3 \leq 1 \cap x_4 \leq 3$

Firing of $t_2 \rightarrow next((M_1, Z_1), t_2) = (M_2, Z_2) = \mathcal{Z}_2$.

Example





$$\mathcal{Z}_1 \cap x_2 \geq 1 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 1 \leq x_2 \leq 1 \\ 0 \leq x_4 \leq 1 \\ 0 \leq x_2 - x_4 \leq 1 \end{array} \right) \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 3 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right) =$$

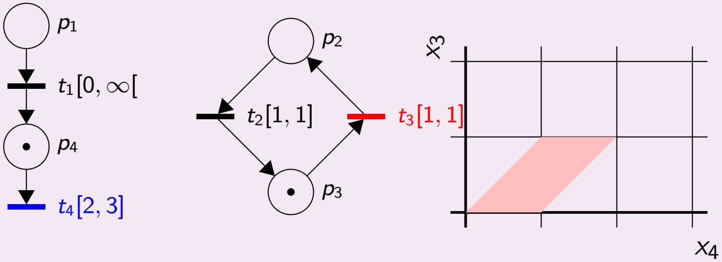
$$\left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right) = \mathcal{Z}_2$$

Compute the futur $\cap\, x_3 \leq 1 \cap x_4 \leq 3$ in canonical form

Firing of $t_3 \rightarrow \mathit{next}((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

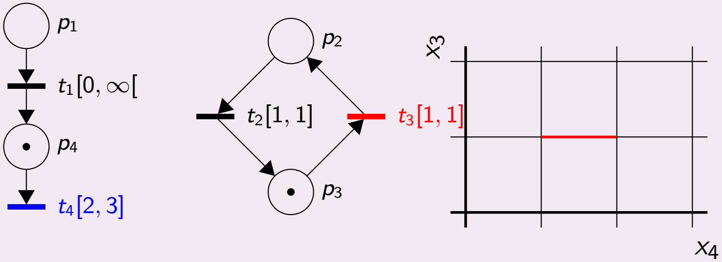
Example



$$\mathcal{Z}_2 = \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right)$$

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example

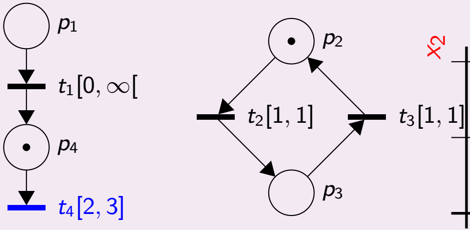


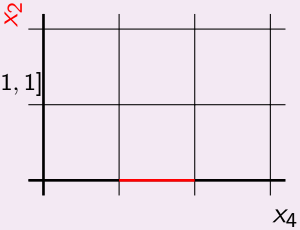
$$Z_2 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{cases}$$

$$(M_2 - {}^{\bullet}t_3 + t_3^{\bullet},\ Z_2 \cap x_3 \geq \alpha_3)$$

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example



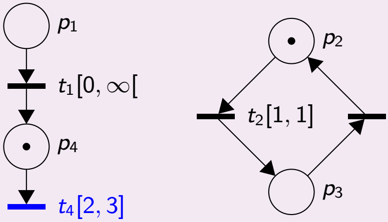


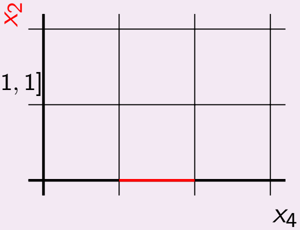
$$Z_2 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 1 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 \end{matrix} \Rightarrow \begin{cases} 0 \leq x_4 \leq 2 \\ x_4 - 1 \leq 1 \\ 1 \leq x_4 \end{cases} \Rightarrow 1 \leq x_4 \leq 2$$

Eliminate x_3 (Fourier-Motzkin method)

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example





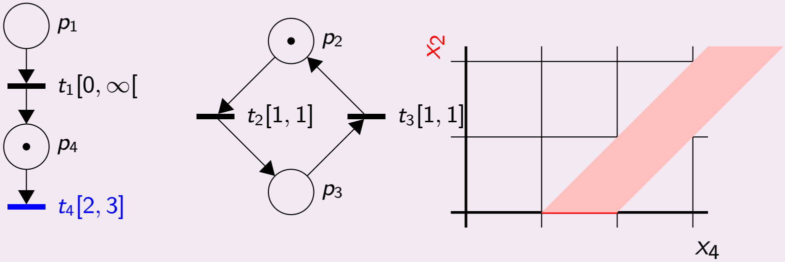
$$Z_2 \cap x_3 \geq 1 = \left\{ \begin{array}{l} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right. \Rightarrow \begin{array}{l} 1 \leq x_3 \\ x_4 - 1 \leq x_3 \end{array} \leq \begin{array}{l} x_3 \leq 1 \\ x_3 \leq x_4 \end{array} \Rightarrow \left\{ \begin{array}{l} 0 \leq x_4 \leq 2 \\ x_4 - 1 \leq 1 \\ 1 \leq x_4 \end{array} \right. \Rightarrow 1 \leq x_4 \leq 2$$

$$\mathcal{Z}_2 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 0 \\ 1 \leq x_4 \leq 2 \\ 1 \leq x_4 - x_2 \leq 2 \end{array} \right)$$

Eliminate x_3 (Fourier-Motzkin method) and add $x_2 = 0$

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example



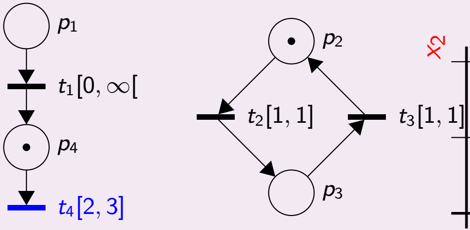
$$Z_2 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 1 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 \end{matrix} \Rightarrow \begin{cases} 0 \leq x_4 \leq 2 \\ x_4 - 1 \leq 1 \\ 1 \leq x_4 \end{cases} \Rightarrow 1 \leq x_4 \leq 2$$

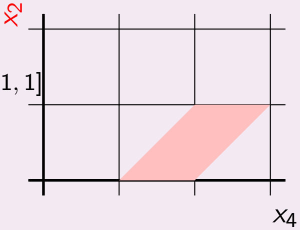
$$\mathcal{Z}_2 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 1 \leq x_4 \leq 2 \\ 1 \leq x_4 - x_2 \leq 2 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \\ 1 \leq x_4 \\ 1 \leq x_4 - x_2 \leq 2 \end{matrix} \right)$$

Compute the futur

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example





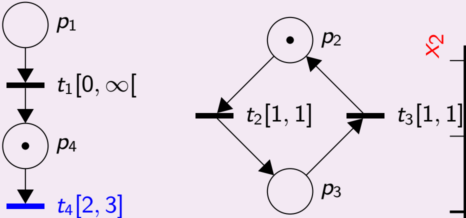
$$Z_2 \cap x_3 \geq 1 = \left\{ \begin{array}{l} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{array} \right. \Rightarrow \begin{array}{l} 1 \leq x_3 \\ x_4 - 1 \leq x_3 \end{array} \leq \begin{array}{l} x_3 \leq 1 \\ x_3 \leq x_4 \end{array} \Rightarrow \left\{ \begin{array}{l} 0 \leq x_4 \leq 2 \\ x_4 - 1 \leq 1 \\ 1 \leq x_4 \end{array} \right. \Rightarrow 1 \leq x_4 \leq 2$$

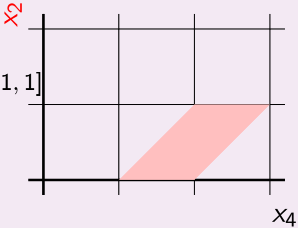
$$\mathcal{Z}_2 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 0 \\ 1 \leq x_4 \leq 2 \\ 1 \leq x_4 - x_2 \leq 2 \end{array} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{array} \right)$$

Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$

Firing of $t_3 \rightarrow next((M_2, Z_2), t_3) = (M_3, Z_3) = \mathcal{Z}_3$.

Example





$$Z_2 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 0 \leq x_4 \leq 2 \\ 0 \leq x_4 - x_3 \leq 1 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 1 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 \end{matrix} \Rightarrow \begin{cases} 0 \leq x_4 \leq 2 \\ x_4 - 1 \leq 1 \\ 1 \leq x_4 \end{cases} \Rightarrow 1 \leq x_4 \leq 2$$

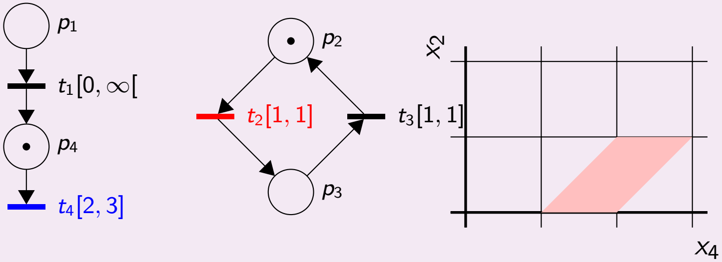
$$\mathcal{Z}_2 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 1 \leq x_4 \leq 2 \\ 1 \leq x_4 - x_2 \leq 2 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{matrix} \right) \Rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{matrix} \right) = \mathcal{Z}_3$$

Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$ in canonical form

Firing of $t_2 \rightarrow next\,((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

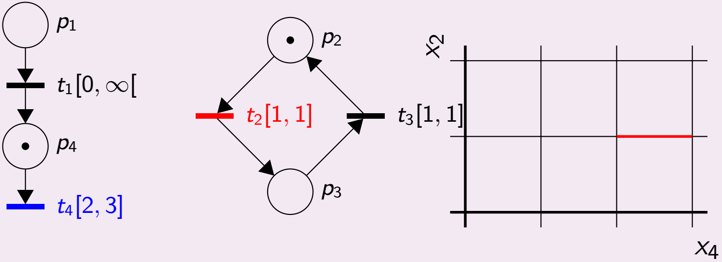
Example



$$\mathcal{Z}_3 = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{array} \right)$$

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Example

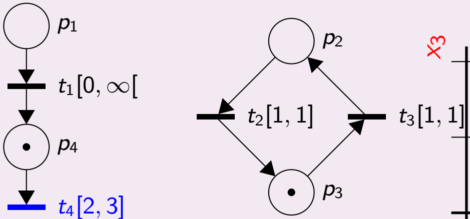


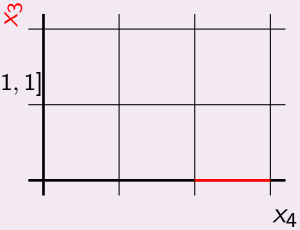
$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases}$$

$$(M_3 - {}^\bullet t_2 + t_2^\bullet,\ Z_3 \cap x_2 \geq \alpha_2)$$

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Example



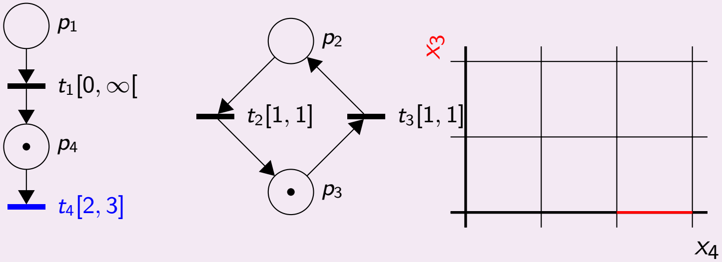


$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_2 \\ x_4 - 2 \leq x_2 \end{matrix} \leq \begin{matrix} x_2 \leq 1 \\ x_2 \leq x_4 - 1 \end{matrix} \Rightarrow \begin{cases} 1 \leq x_4 \leq 3 \\ x_4 - 2 \leq 1 \\ 1 \leq x_4 - 1 \end{cases} \Rightarrow 2 \leq x_4 \leq 3$$

Eliminate x_2 (Fourier-Motzkin method)

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Example



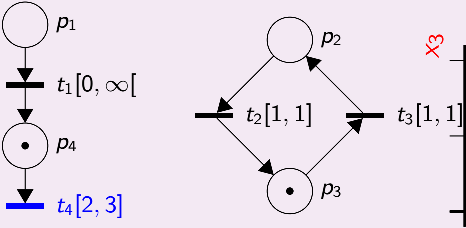
$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_2 \\ x_4 - 2 \leq x_2 \end{matrix} \leq \begin{matrix} x_2 \leq 1 \\ x_2 \leq x_4 - 1 \end{matrix} \Rightarrow \begin{cases} 1 \leq x_4 \leq 3 \\ x_4 - 2 \leq 1 \\ 1 \leq x_4 - 1 \end{cases} \Rightarrow 2 \leq x_4 \leq 3$$

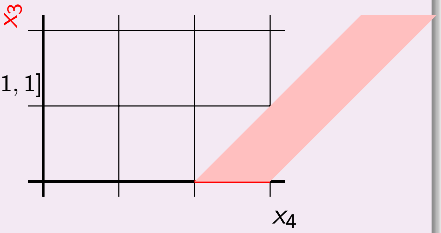
$$\mathcal{Z}_3 \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 0 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right)$$

Eliminate x_2 (Fourier-Motzkin method) and add $x_3 = 0$

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Example



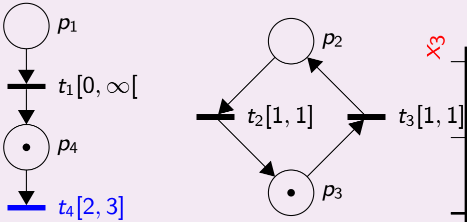


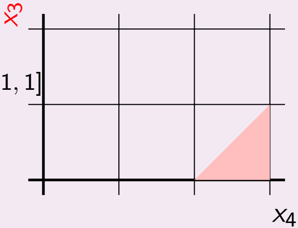
$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_2 \\ x_4 - 2 \leq x_2 \end{matrix} \leq \begin{matrix} x_2 \leq 1 \\ x_2 \leq x_4 - 1 \end{matrix} \Rightarrow \begin{cases} 1 \leq x_4 \leq 3 \\ x_4 - 2 \leq 1 \\ 1 \leq x_4 - 1 \end{cases} \Rightarrow 2 \leq x_4 \leq 3$$

$$\mathcal{Z}_3 \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 0 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \\ 2 \leq x_4 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right)$$

Compute the futur

Example





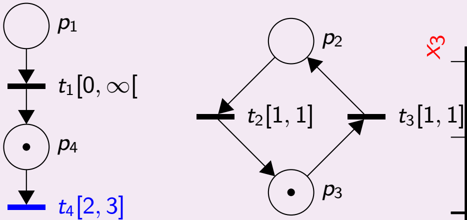
$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_2 \\ x_4 - 2 \leq x_2 \end{matrix} \leq \begin{matrix} x_2 \leq 1 \\ x_2 \leq x_4 - 1 \end{matrix} \Rightarrow \begin{cases} 1 \leq x_4 \leq 3 \\ x_4 - 2 \leq 1 \\ 1 \leq x_4 - 1 \end{cases} \Rightarrow 2 \leq x_4 \leq 3$$

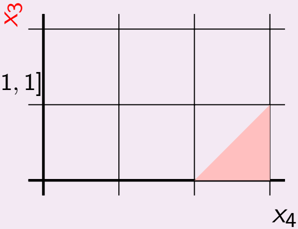
$$\mathcal{Z}_3 \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 0 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right)$$

Compute the futur $\cap\, x_3 \leq 1 \cap x_4 \leq 3$

Firing of $t_2 \rightarrow next((M_3, Z_3), t_2) = (M_4, Z_4) = \mathcal{Z}_4$.

Example





$$Z_3 \cap x_2 \geq 1 = \begin{cases} 1 \leq x_2 \leq 1 \\ 1 \leq x_4 \leq 3 \\ 1 \leq x_4 - x_2 \leq 2 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_2 \\ x_4 - 2 \leq x_2 \end{matrix} \leq \begin{matrix} x_2 \leq 1 \\ x_2 \leq x_4 - 1 \end{matrix} \Rightarrow \begin{cases} 1 \leq x_4 \leq 3 \\ x_4 - 2 \leq 1 \\ 1 \leq x_4 - 1 \end{cases} \Rightarrow 2 \leq x_4 \leq 3$$

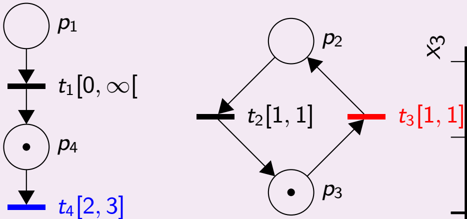
$$\mathcal{Z}_3 \xrightarrow{t_2} \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 0 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{matrix} \right) = \mathcal{Z}_4$$

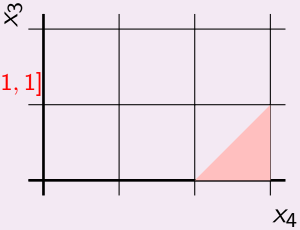
Compute the futur $\cap\, x_3 \leq 1 \cap x_4 \leq 3$ in canonical form

Firing of $t_3 \rightarrow next\,((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example

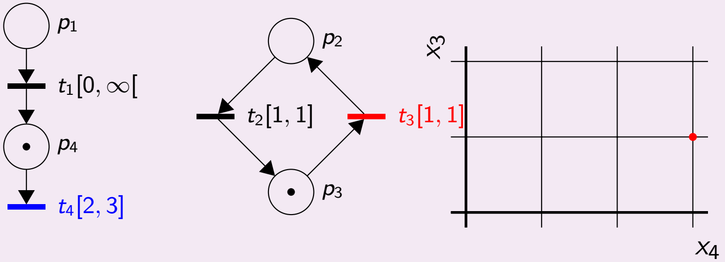




$$\mathcal{Z}_4 = \left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{array}{l} 0 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{array} \right)$$

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example

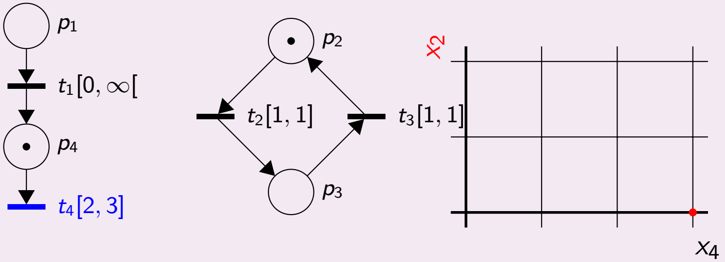


$$Z_4 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{cases}$$

$$(M_4 - {}^\bullet t_3 + t_3^\bullet,\ Z_4 \cap x_3 \geq \alpha_3)$$

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example

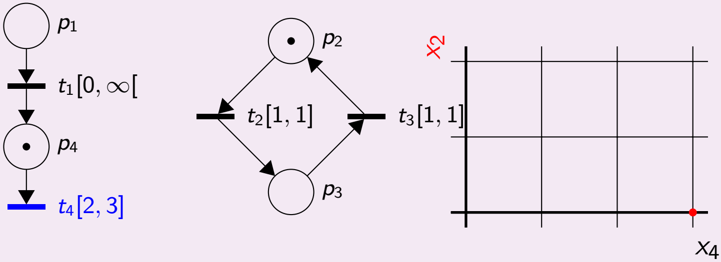


$$Z_4 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 3 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 - 2 \end{matrix} \Rightarrow \begin{cases} 2 \leq x_4 \leq 3 \\ x_4 - 3 \leq 1 \\ 1 \leq x_4 - 2 \end{cases} \Rightarrow 3 \leq x_4 \leq 3$$

Eliminate x_3 (Fourier-Motzkin method)

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example



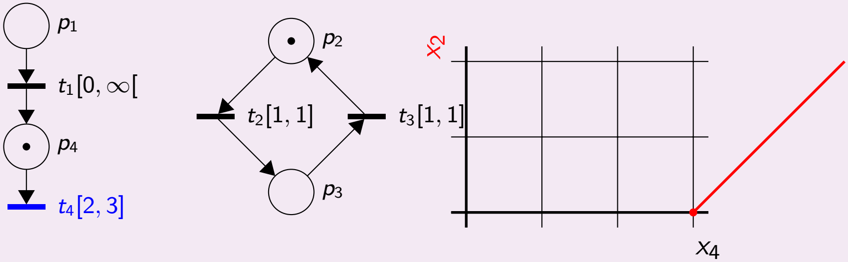
$$Z_4 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 3 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 - 2 \end{matrix} \Rightarrow \begin{cases} 2 \leq x_4 \leq 3 \\ x_4 - 3 \leq 1 \\ 1 \leq x_4 - 2 \end{cases} \Rightarrow 3 \leq x_4 \leq 3$$

$$\mathcal{Z}_4 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right)$$

Eliminate x_3 (Fourier-Motzkin method) and add $x_2 = 0$

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example



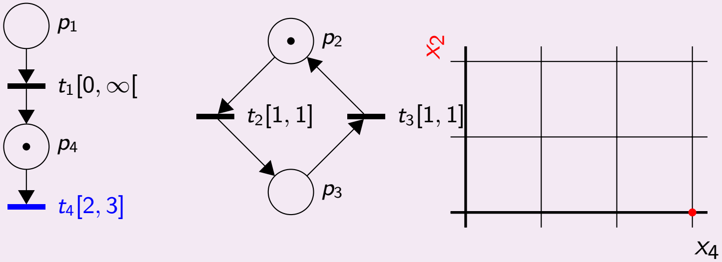
$$Z_4 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 3 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 - 2 \end{matrix} \Rightarrow \begin{cases} 2 \leq x_4 \leq 3 \\ x_4 - 3 \leq 1 \\ 1 \leq x_4 - 2 \end{cases} \Rightarrow 3 \leq x_4 \leq 3$$

$$\mathcal{Z}_4 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \\ 3 \leq x_4 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right)$$

Compute the futur

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example



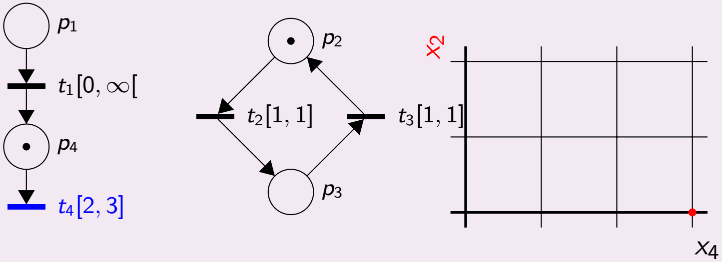
$$Z_4 \cap x_3 \geq 1 = \begin{cases} 1 \leq x_3 \leq 1 \\ 2 \leq x_4 \leq 3 \\ 2 \leq x_4 - x_3 \leq 3 \end{cases} \Rightarrow \begin{matrix} 1 \leq x_3 \\ x_4 - 3 \leq x_3 \end{matrix} \leq \begin{matrix} x_3 \leq 1 \\ x_3 \leq x_4 - 2 \end{matrix} \Rightarrow \begin{cases} 2 \leq x_4 \leq 3 \\ x_4 - 3 \leq 1 \\ 1 \leq x_4 - 2 \end{cases} \Rightarrow 3 \leq x_4 \leq 3$$

$$\mathcal{Z}_4 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 1 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right)$$

Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$

Firing of $t_3 \rightarrow next((M_4, Z_4), t_3) = (M_5, Z_5) = \mathcal{Z}_5$.

Example



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$$\mathcal{Z}_4 \xrightarrow{t_3} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right) \rightarrow \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 1 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right) = \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{matrix} 0 \leq x_2 \leq 0 \\ 3 \leq x_4 \leq 3 \\ 3 \leq x_4 - x_2 \leq 3 \end{matrix} \right) = \mathcal{Z}_5$$

Compute the futur $\cap\, x_2 \leq 1 \cap x_4 \leq 3$ in canonical form

Terminaison

Calcul de l'espace d'états

Theorem

L'algorithme converge pour les réseaux de Petri temporels :

- *bornés*
- $\beta : T \rightarrow \mathbb{Q}^{+}$

Calcul de l'espace d'états

Theorem

L'algorithme converge pour les réseaux de Petri temporels :

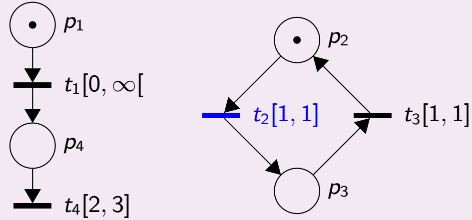
- *bornés*
- $\beta : T \rightarrow \mathbb{Q}^+$

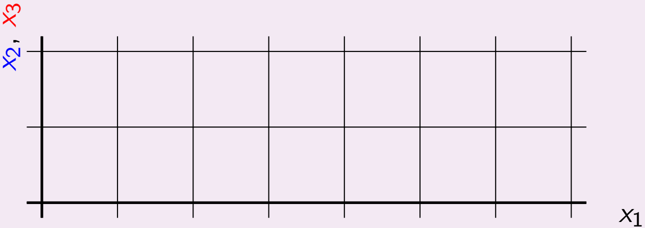
Problème

Terminaison pour $\beta : T \rightarrow \mathbb{Q}^+ \cup \{\infty\}$?

Calcul de l'espace d'états

Example (Prise en compte de l'infini)





Calcul de l'espace d'états

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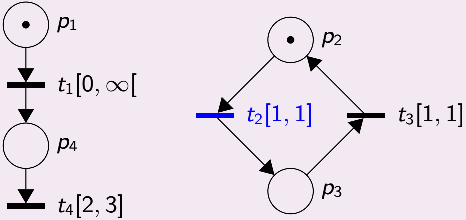
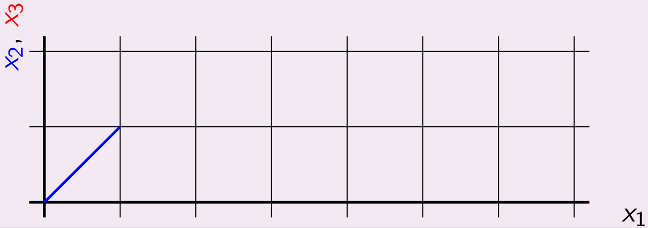
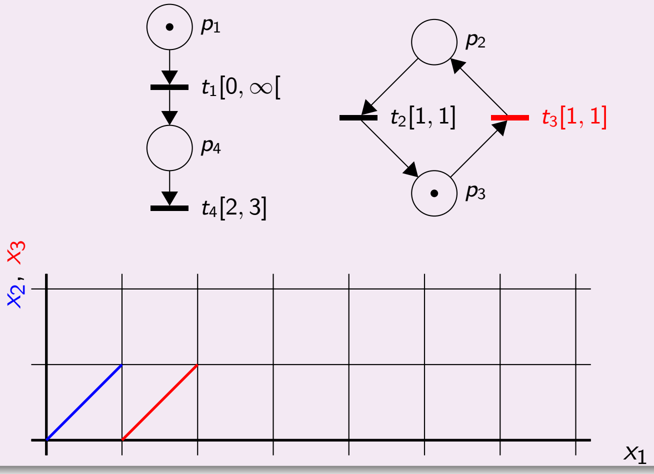




t_2

Calcul de l'espace d'états

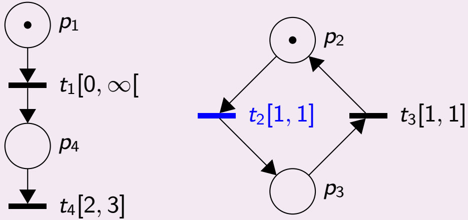
Example (Prise en compte de l'infini)

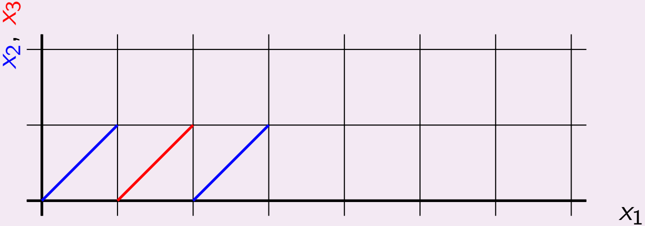


t_2, t_3

Calcul de l'espace d'états

Example (Prise en compte de l'infini)

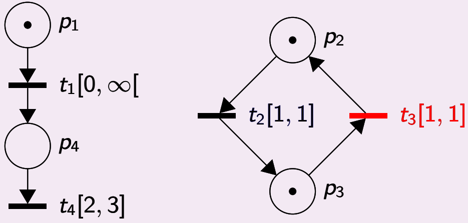


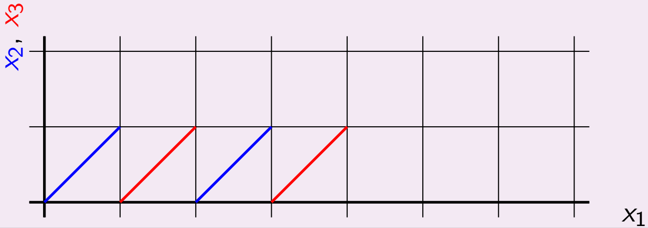


t_2, t_3, t_2

Calcul de l'espace d'états

Example (Prise en compte de l'infini)

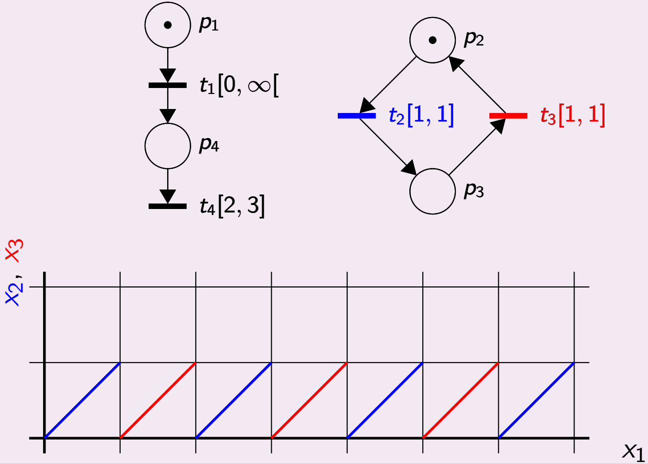




$$t_2, t_3, t_2, t_3 \ldots$$

Calcul de l'espace d'états

Example (Prise en compte de l'infini)



$$(t_2, t_3)^*$$

Transition de type $[a, \infty[$:

- Information importante : $x \geq a$

→ Utilisation d'un opérateur d'approximation *k-approx*

- Choix de k :

$$k = max_{t \in T \mid \beta(t) \neq \infty} (\alpha(t), \beta(t))$$

Calcul de l'espace d'état

Étape 4

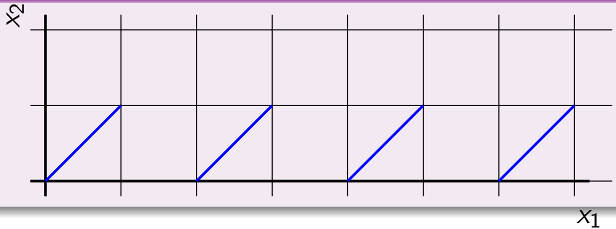
Appliquer l'opérateur à chaque calcul de successeur

Calcul de l'espace d'état

Étape 4

Appliquer l'opérateur à chaque calcul de successeur

Example

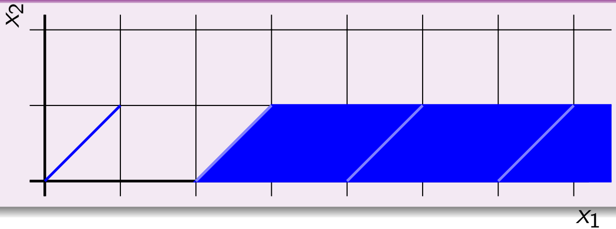


$$(M_i, Z_i) = (M_{i-1} - {}^\bullet t + t^\bullet, \texttt{k-approx}(zone))$$

Calcul de l'espace d'état
Étape 4

Appliquer l'opérateur à chaque calcul de successeur

Example



$$(M_i, Z_i) = (M_{i-1} -^{\bullet} t + t^{\bullet}, \texttt{k-approx}(zone))$$

Calculs de l'espace d'états

Theorem

L'algorithme du graphe des zones avec k-approximation est exacte vis à vis de l'accessibilité d'un marquage pour les réseaux de Petri temporels bornés à dates de tir au plus tard dans l'ensemble $\mathbb{Q}^+ \cup \{\infty\}$.

Calculs de l'espace d'états

Theorem

L'algorithme du graphe des zones avec k-approximation est exacte vis à vis de l'accessibilité d'un marquage pour les réseaux de Petri temporels bornés à dates de tir au plus tard dans l'ensemble $\mathbb{Q}^+ \cup \{\infty\}$.

Remarques :

- Implémentation efficace avec des DBM
- Implémentée dans `UPPAAL` pour les TA et dans `Roméo` pour les TPN
- Une autre approche en considérant l'écoulement du temps par décroissance des intervalles au lieu des horloges : le graphe des classes d'état

Plan

Outils

Outils pour le Calcul de l'espace d'états

Zones ou Classes

- Graphe des classes :
  - TINA [Berthomieu et al., 2004]
  - ROMÉO [Lime et al., 2009],
  - ORIS [Bucci et al., 2010]
- Graphe des zones
  - UPPAAL [Larsen et al., 1997]
  - ROMÉO [Lime et al., 2009]

- TINA : `https://projects.laas.fr/tina/`
- ROMÉO : `https://romeo.ls2n.fr`
- ORIS : `https://www.oris-tool.org`

Merci de votre attention

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